

Fundamental strings and black holes

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ABSTRACT: We propose a black hole thermodynamic description of highly excited charged and uncharged perturbative string states in 3+1 dimensional type II and 4+1 dimensional heterotic string theory. We also discuss the generalization to extremal and non-extremal black holes carrying magnetic charges.

KEYWORDS: Superstrings and Heterotic Strings, Black Holes in String Theory, Conformal Field Models in String Theory.

JHEP01(2007)071

Contents

1.	Introduction	1
2.	Type II strings	3
2.1	Type II strings as black holes	3
2.2	Charged strings as black holes	7
2.3	States carrying electric and magnetic charges	9
3.	Heterotic strings	11
3.1	Thermodynamic description of highly excited bosonic strings	11
3.2	Heterotic strings as black holes	12
3.3	Charged heterotic strings as black holes	14
3.4	States carrying electric and magnetic charges	14
4.	Discussion	16

1. Introduction

In quantum gravity in asymptotically flat 3 + 1 dimensional spacetime, generic high energy states with vanishing charge¹ are believed to be described by the Schwarzschild geometry. Their entropy is expected to be given by the Bekenstein-Hawking (BH) formula

$$S = A/4G_N , \tag{1.1}$$

with $A = 4\pi R_h^2$ the area of the horizon, R_h the Schwarzschild radius and G_N the Newton constant (see e.g. [1] for a review).

This description is thermodynamic in nature. The Euclidean black hole solution contributes to the canonical free energy at a temperature equal to the Hawking temperature of the black hole. The Minkowski solution can be thought of as an average over all states with energy equal to the mass of the black hole. Thus, it corresponds to the microcanonical ensemble. The entropy — energy relation implied by (1.1) is $S = 4\pi G_N M^2$. This is the leading term in an asymptotic expansion in inverse energy. Corrections are due to quantum and other effects.

The problem of providing a statistical interpretation to black hole thermodynamics has received a lot of attention over the years and much progress has been achieved (see e.g. [1 – 4] for reviews). In particular, for gravity in asymptotically anti-de-Sitter spacetime,

¹There are generalizations to states with charge, angular momentum, as well as to other dimensions of spacetime.

which is dual to a field theory [5], the entropy of large AdS-Schwarzschild black holes is expected (and in some examples was verified) to agree with the high energy density of states of the dual field theory. In other cases, such as large Schwarzschild black holes in asymptotically flat spacetime, the nature of the “microstates” that lead to the BH entropy is not understood.

String theory contains gravity, so the above discussion applies to it. When the string coupling g_s is small, the theory has two widely separated scales. One is l_p , at which quantum gravity effects become important. The other is $l_s = \sqrt{\alpha'} = 1/M_s$, at which string corrections become important. For example, in $3 + 1$ dimensions one has $l_p = g_s l_s$, so $l_p \ll l_s$ at weak coupling.

In weakly coupled string theory, the Schwarzschild solution describes the thermodynamics for energies $E \gg M_s/g_s^2$. In this regime the Schwarzschild radius is large ($R_h \gg l_s$) and gravity is reliable, but the spectrum of the theory is not understood. On the other hand, for El_s large but finite in the limit $g_s \rightarrow 0$ the spectrum is known, and one can study the corresponding statistical mechanics using standard tools. To leading order in g_s , the free energy is given by the string partition sum with Euclidean time compactified on a circle of circumference $\beta = 1/T$, evaluated on a worldsheet torus [6–9]. This approach is statistical in nature, since the partition sum is obtained by tracing over microstates.

A natural question is whether there is a description of the perturbative string spectrum in terms of black hole thermodynamics. It is reasonable to expect that such a description exists since for $El_s \gg 1$ the free string entropy is large and a thermodynamic description should be appropriate. Indeed, for some classes of perturbative heterotic string states with mass equal to charge, thermodynamic descriptions in terms of extremal black holes were proposed before [10–14].

The main purpose of this note is to propose a thermodynamic description for a class of perturbative string states with generic mass and charges in $3 + 1$ dimensional type II and $4 + 1$ dimensional heterotic string theory. According to this description, as one approaches a highly excited fundamental string of mass M , the angular sphere shrinks and decouples from the radial direction and time. The latter are described by a two dimensional geometry with asymptotically flat metric and linear dilaton in the radial direction. The full near-horizon geometry is described by the (charged or uncharged) two dimensional black hole [15–20]. We show that the thermodynamic entropy of the black hole matches that of the corresponding perturbative string states to leading order in M_s/M and any mass to charge ratio.

The above construction seems to be special to $3 + 1$ dimensions in type II and $4 + 1$ dimensions in heterotic string theory. In other dimensions one should also be able to replace highly excited strings by geometries, but these geometries might be more complicated. An important feature of our proposal is the decoupling of the angular sphere from the radial coordinate and time in the near-horizon geometry. This decoupling probably does not occur in general and the near-horizon geometry is described by a highly curved background which involves all d dimensions of spacetime.

We also discuss the generalization of the above construction to the case where in addition to the electric charges carried by perturbative strings we turn on magnetic charges

associated with Neveu-Schwarz fivebranes and Kaluza-Klein monopoles. We construct the exact worldsheet background corresponding to systems with generic values of these four charges and mass, and study their thermodynamics.

2. Type II strings

We start with type II string theory on

$$\mathbb{R}^{3,1} \times \mathcal{C}_6, \tag{2.1}$$

where \mathcal{C}_6 is a compact manifold. The sigma model on \mathcal{C}_6 is a unitary $N = 1$ superconformal field theory with central charge $c = 9$ and a discrete spectrum of scaling dimensions starting at zero. We are interested in the thermodynamics of highly excited perturbative string states in this background. In this section we will provide such a description, first for uncharged states and then for states carrying up to two charges associated with a circle in \mathcal{C}_6 .

2.1 Type II strings as black holes

One way to approach the problem is to start with a large Schwarzschild black hole, with Schwarzschild radius $R_h \gg l_s$, and ask what happens when its mass decreases. For the Schwarzschild solution, one has $R_h = 2G_N M$, so as M decreases R_h does as well. Eventually it becomes of order l_s and string corrections become important. Formally, this happens when $M \sim M_s/g_s^2$, which is the transition region between the black hole and string regimes [21–24]. Below that mass, the size of the typical state is larger than the horizon radius, but one can still hope to use a black hole picture to study the thermodynamics.

As $G_N M/l_s \rightarrow 0$ we expect the BH temperature to approach the Hagedorn one, and black hole thermodynamics to match smoothly to perturbative string thermodynamics. In particular, for $M \gg M_s$ and $g_s \rightarrow 0$ the black hole entropy should behave like

$$S = \beta_H M, \tag{2.2}$$

with

$$\beta_H = 2\sqrt{2\pi}l_s \tag{2.3}$$

the inverse Hagedorn temperature of perturbative type II strings.

This behavior should be reproduced by the α' corrected near-horizon Schwarzschild geometry. To find this geometry by systematically including these corrections is a formidable task, both because the perturbative corrections to the Einstein equations are known only to the first few orders in α' , and because non-perturbative effects might be important. We will next make a proposal for its form.

For large values of the radial coordinate r , the space around a small black hole is flat (\mathbb{R}^3). As r decreases, the radius of the angular two-sphere S^2 shrinks. We will *assume* that near the horizon the geometry factorizes into a 1 + 1 dimensional background describing (t, r) and a two-dimensional one for the S^2 . The main motivation for this assumption is that this is expected to happen for certain charged extremal four dimensional black holes

that preserve some supersymmetry (see e.g. [25] for a recent review). As we will see later, it is then natural to expect that it happens for non-extremal and uncharged black holes as well.

At any rate, under the above assumption the worldsheet CFT corresponding to S^2 is trivial for the following reason. By construction, this CFT must have an $SO(3)$ global symmetry. This implies the existence of a worldsheet current (J^a, \bar{J}^a) , with $a = 1, 2, 3$, in the adjoint of $SO(3)$, satisfying the conservation equation

$$\bar{\partial}J^a + \partial\bar{J}^a = 0 . \tag{2.4}$$

Since the worldsheet theory on S^2 is a unitary CFT with a discrete spectrum, one can show in general that J^a and \bar{J}^a must be separately conserved, i.e. $\bar{\partial}J^a = \partial\bar{J}^a = 0$. If one of the currents is non-zero, the other should be non-zero as well since the worldsheet CFT corresponding to the small black hole background should be left-right symmetric. The symmetry is thus enhanced to $SU(2)_L \times SU(2)_R$ with J^a and \bar{J}^a satisfying the corresponding affine Lie algebra relations. As is well known, backgrounds with such symmetries correspond geometrically to S^3 rather than S^2 , and contain fluxes of the NS B-field through the sphere, which makes them inappropriate here. The currents J^a and \bar{J}^a must thus vanish, so the simplest possibility is that the worldsheet theory corresponding to S^2 is trivial.

We conclude that the near-horizon geometry of the small black hole contains only the radial direction and time. The corresponding worldsheet CFT should have the same central charge as that describing the flat spacetime at infinity. Since time translation invariance is a symmetry, the simplest possibility is that the near-horizon geometry contains a dilaton which is asymptotically linear in the radial direction ϕ , a function of r that corresponds near the boundary of this geometry to a canonically normalized scalar field.

The slope of the dilaton, Q , determines the central charge of the worldsheet theory for ϕ via the relation

$$c_\phi = 1 + 3Q^2 . \tag{2.5}$$

Comparing the central charge of the theory of (t, ϕ) to that of $\mathbb{R}^{3,1}$ fixes Q . One finds that $Q = 1$, such that the total central charge of (t, ϕ) and their worldsheet superpartners is equal to six.

One can think of the appearance of the linear dilaton as an analog of gravitational RG flow. It is well known (see e.g. [26, 27]) that if one couples a two dimensional theory which interpolates between UV and IR fixed points along an RG flow to worldsheet gravity, the RG flow proceeds as a function of the Liouville coordinate ϕ . The boundary of space, $\phi \rightarrow \infty$, where the string coupling goes to zero, corresponds to the UV fixed point, while $\phi \rightarrow -\infty$ corresponds to the IR fixed point. In our case, the analog of the Liouville coordinate is the radial coordinate r or ϕ . On its own, the sigma model on the angular two-sphere in \mathbb{R}^3 naturally goes to smaller central charge, and is massive in the infrared. The radial direction compensates by developing a non-trivial dilaton and increasing its central charge.

To summarize, we propose that the asymptotic form of the near-horizon geometry of small black holes (i.e. those with masses in the perturbative string regime) is

$$\mathbb{R}_t \times \mathbb{R}_\phi \times \mathcal{C}_6 . \tag{2.6}$$

The dilaton is linear in the radial coordinate ϕ such that this background is critical. All excitations in the near-horizon geometry (2.6) are singlets of $\text{SO}(3)$ (i.e. s-waves). A related fact is that if the full background (2.1) preserves some supersymmetry, only half of the generators are visible in the near-horizon geometry (2.6). The other half act trivially on all degrees of freedom there, like the generators of $\text{SO}(3)$.

The small black hole must correspond to a two dimensional black hole in $\mathbb{R}_t \times \mathbb{R}_\phi$. The unique solution to the string equations of motion with the right properties is the $\text{SL}(2, \mathbb{R})/\text{U}(1)$ black hole [15–17], which is described by the metric and dilaton (here we set $\alpha' = 2$)

$$\begin{aligned} ds^2 &= f^{-1}d\phi^2 - fdt^2, & f &= 1 - \frac{2M}{\rho}, \\ Qe^{-2\Phi} &= Qe^{Q\phi} = \rho . \end{aligned} \tag{2.7}$$

The dilaton is asymptotically linear in ϕ , and approaches a constant,

$$e^{-2\Phi_h} = \frac{2M}{Q}, \tag{2.8}$$

at the horizon. The level of $\text{SL}(2)$, k , is in general related to the linear dilaton slope via the relation

$$Q = \sqrt{\frac{2}{k}} . \tag{2.9}$$

Note that k is the total level of $\text{SL}(2, \mathbb{R})$. The worldsheet theory is superconformal; it consists of a bosonic $\text{SL}(2, \mathbb{R})$ WZW model of level $k + 2$, and three free fermions in the adjoint representation that contribute -2 to the level. In our case $Q = 1$, so $k = 2$. This value lies above the transition of [28, 20], so the black hole is a normalizable state in the near-horizon geometry.

The Euclidean black hole is obtained by Wick rotating $t \rightarrow it$ in (2.7). This gives rise to a cigar geometry, and one can read off the inverse temperature of the black hole from the circumference of Euclidean time at infinity. The result is

$$\beta_H = 2\pi l_s \sqrt{k}, \tag{2.10}$$

which corresponds to a Hagedorn entropy at high energies $E \gg M_s$,

$$S_{\text{BH}} = \beta_H E . \tag{2.11}$$

This result is valid for all k , despite the fact that (2.7) was obtained by solving the equations of motion of dilaton gravity, which are only valid for large k (or small Q). This can be shown by analyzing the exact spectrum of the coset theory $\text{SL}(2, \mathbb{R})/\text{U}(1)$ algebraically.

From the sigma model point of view, it follows from the fact that the coset preserves $N = 2$ supersymmetry, which leads to non-renormalization of the background (2.7).

Altogether, we conclude that the small black hole that provides a thermodynamic description of uncharged fundamental string states with mass $M \gg M_s$ in $\mathbb{R}^{3,1} \times \mathcal{C}_6$ is

$$\frac{\text{SL}(2, \mathbb{R})_2}{\text{U}(1)} \times \mathcal{C}_6 . \tag{2.12}$$

This black hole has a Hagedorn entropy (2.11). Its Hagedorn temperature (2.10) coincides² with that of perturbative fundamental strings (2.3).

The assertion that perturbative string states develop a linear dilaton throat in their vicinity sounds at first sight surprising. Note that this is only expected to occur for states with sufficiently small mass. The black hole geometry (2.7), in which the dilaton decreases as one moves away from the horizon, attaches to flat space at the place where e^Φ reaches its asymptotic value, g_s . In order for it to be valid in a significant range of distances, the string coupling at the horizon, (2.8), must be much larger than the asymptotic coupling, or:

$$M \ll \frac{M_s}{g_s^2} . \tag{2.13}$$

Thus, as one would expect, our description is only valid well below the string/black hole correspondence region of [21–24].

From the point of view of the full geometry, the throat (2.7) occupies a string size region around $r = 0$. A simple way to see that is to note that in (2.7) the two-sphere has already disappeared. Thus, (2.7) must attach to the rest of the geometry in a region where the size of the two-sphere is of the order of the string scale. Another, heuristic, way to estimate the radial size of the throat is to note that the geometry (2.7) is reminiscent of the Schwarzschild one, with the replacement $\rho \rightarrow r/g_s^2$. In the coordinate ρ the transition between the linear dilaton throat and flat spacetime occurs at $\rho \sim 1/g_s^2$, which is equivalent to $r \simeq 1$.

As the mass M of the string state increases towards $M \sim M_s/g_s^2$, the linear dilaton region shrinks and eventually disappears. On the other hand, as M/M_s decreases the throat formally becomes longer, but the string coupling at the horizon grows and for $M \simeq M_s$ the theory becomes strongly coupled. This is natural from the point of view of string thermodynamics since the entropy of the corresponding fundamental string states is small and one expects large fluctuations in the thermal description.

One can also ask what happens for other dimensions of spacetime. The general considerations above suggest that one should still be able to replace fundamental strings with $M \gg M_s$ by a geometry with a horizon. However, in this case the description of the near-horizon region by an $\text{SL}(2, \mathbb{R})/\text{U}(1)$ black hole is inconsistent with the free string entropy. We suspect that the origin of the problem is the assumption of decoupling of the angular sphere S^{d-2} from the radial coordinate and time. In general, there is no motivation for assuming this decoupling, and without it it is difficult to determine the near-horizon

²The fact that the Little String Theory entropy agrees with the perturbative string one for $k = 2$, as well as some other relations between the two problems, were pointed out in [29].

geometry. The situation for states with non-zero angular momentum in 3 + 1 dimensions is also more complicated due to the absence of SO(3) symmetry in the corresponding black hole solution.

2.2 Charged strings as black holes

In this subsection we will generalize the discussion of the previous subsection to fundamental string states which carry momentum n and winding w around a circle of radius R . Thus, we will take the geometry (2.1) to have the form

$$\mathbb{R}^{3,1} \times S^1 \times \mathcal{C}_5 . \tag{2.14}$$

The left and right-moving momentum of the string on the S^1 is given by

$$(q_L, q_R) = \left(\frac{n}{R} + \frac{wR}{\alpha'} , \frac{n}{R} - \frac{wR}{\alpha'} \right) . \tag{2.15}$$

The mass-shell condition is

$$\alpha' M^2 = 4N_L + \alpha' q_L^2 = 4N_R + \alpha' q_R^2 , \tag{2.16}$$

where N_L, N_R are the left and right-moving oscillator levels.

For large N_L and/or N_R , the entropy of free strings with mass M and charges (q_L, q_R) is given by

$$S = 2\pi\sqrt{2} \left(\sqrt{N_L} + \sqrt{N_R} \right) = \pi l_s \sqrt{2} \left(\sqrt{M^2 - q_L^2} + \sqrt{M^2 - q_R^2} \right) . \tag{2.17}$$

For $q_L = q_R = 0$ we saw in the previous subsection that the black hole background (2.12) provides a thermodynamic description of these states. Black holes with generic (q_L, q_R) are obtained by adding a circle to the uncharged black hole, performing a boost along it, followed by T-duality and another boost. This leads [30] to the background

$$\frac{\text{SL}(2, \mathbb{R})_2 \times \text{U}(1)}{\text{U}(1)} \times \mathcal{C}_5 . \tag{2.18}$$

The charge to mass ratio of the black hole determines the way the U(1) in the denominator is embedded in $\text{SL}(2, \mathbb{R})_2 \times \text{U}(1)$. Denoting by (J^3, \bar{J}^3) the left and right-moving currents in a space-like Cartan subalgebra of $\text{SL}(2, \mathbb{R})_L \times \text{SL}(2, \mathbb{R})_R$, and by (J, \bar{J}) the currents corresponding to the U(1) factor in the numerator of (2.18), the left and right components of the gauged U(1) current (J_L, J_R) are given by

$$\begin{aligned} J_L &= J^3 \cos \alpha_L + J \sin \alpha_L , \\ J_R &= \bar{J}^3 \cos \alpha_R + \bar{J} \sin \alpha_R , \end{aligned} \tag{2.19}$$

where

$$\begin{aligned} \sin \alpha_L &= \frac{q_L}{M} , \\ \sin \alpha_R &= \frac{q_R}{M} . \end{aligned} \tag{2.20}$$

In the special case $q_L = q_R = 0$, we have $\alpha_L = \alpha_R = 0$, and the background (2.18) reduces to the uncharged two dimensional black hole (2.7), (2.12), with $\mathcal{C}_6 = S^1 \times \mathcal{C}_5$.

For generic (q_L, q_R) , the $\frac{\text{SL}(2, \mathbb{R}) \times \text{U}(1)}{\text{U}(1)}$ factor in (2.18) describes time, the radial coordinate of the four dimensional charged black hole and the circle under which the fundamental strings are charged. The angular two-sphere decouples in the near-horizon region, as in the uncharged case.

The three dimensional background in (2.18) is a special case of a more general class of charged black holes of the form

$$\frac{\text{SL}(2, \mathbb{R})_k \times \text{U}(1)}{\text{U}(1)} . \tag{2.21}$$

The geometry and thermodynamics of these black holes were studied in [20]. Their entropy is

$$S_{\text{BH}} = \pi l_s \sqrt{k} \left(\sqrt{M^2 - q_L^2} + \sqrt{M^2 - q_R^2} \right) . \tag{2.22}$$

As in the uncharged case, one can obtain (2.22) by analyzing the background (2.21) algebraically. One can also describe this background as a solution to dilaton gravity coupled to gauge fields and use it to study the thermodynamics. Apriori this analysis is only valid for large k , but in fact it is expected to be exact, as for $q_L = q_R = 0$.

For $k = 2$, (2.22) is equal to the free string entropy (2.17) and we propose that the background (2.18) is in fact the near-horizon geometry of the corresponding string states. Like (2.17), (2.22) is valid to leading order in M_s/M and for arbitrary charge to mass ratio α_L, α_R (2.20).

In general, the black hole (2.18) is non-extremal and breaks all supersymmetry. In some special cases, which correspond to extremal black holes, part of the supersymmetry is preserved by the solution. In particular, for

$$M = |q_R| \tag{2.23}$$

and generic q_L the solution preserves a quarter of the supercharges of the background (2.6) and provides a thermodynamic description of the corresponding 1/4 BPS Dabholkar-Harvey states [31, 32]. In this case, α_L in (2.20) is generic, while $|\sin \alpha_R| = 1$. Thus, the right-moving component of (2.19) lies purely in the S^1 in (2.14), (2.18), while J_L acts both on the $\text{SL}(2, \mathbb{R})$ and the $\text{U}(1)$ in (2.18).

For large k , one can think of (2.21) as a sigma model on $AdS_2 \times S^1$, whose properties can be obtained from the results of appendix C of [20]. The radii of the AdS space and the circle are given by

$$R_{\text{AdS}} = \frac{l_s}{2} \sqrt{k} , \tag{2.24}$$

and

$$R = l_s \sqrt{\left| \frac{n}{w} \right|} . \tag{2.25}$$

The gauge fields associated with G and B on the circle have constant field strengths on AdS_2 (proportional to n and w). The two dimensional dilaton takes the value

$$\frac{1}{g_2^2} = \sqrt{k|nw|} . \tag{2.26}$$

In the case of interest to (2.18), $k = 2$, the background (2.21) is highly stringy, but as in the other cases discussed above one can still formally continue the sigma model picture to this regime, and the results (2.24), (2.25) do not receive α' corrections. String loop corrections are small for $|nw| \gg 1$, the regime of interest.

A further restriction to the case

$$M = q_L = -q_R, \tag{2.27}$$

corresponding to a string with winding but no momentum around S^1 , leads to a black hole that preserves half of the supersymmetry. From (2.19), (2.20) we see that in this case the gauging does not act on the $SL(2, \mathbb{R})$ in (2.18), and the resulting background is

$$SL(2, \mathbb{R})_2 \times \mathcal{C}_5. \tag{2.28}$$

The three dimensional string coupling can be obtained by combining (2.25), (2.26):

$$\frac{1}{g_3^2} = \sqrt{k}|w| = \sqrt{2}|w|. \tag{2.29}$$

Thus, string theory in the background (2.28) is weakly coupled for large $|w|$, and can be studied using standard perturbative string theory techniques.

2.3 States carrying electric and magnetic charges

So far we focused on states which carry only electric charges (2.15) on the S^1 in (2.14). In this subsection we generalize the discussion to states that carry magnetic charges as well. To do that, we take the compact manifold \mathcal{C}_5 in (2.14) to have the form $\mathcal{C}_5 = \tilde{S}^1 \times \mathcal{C}_4$, so that the geometry (2.14) is

$$\mathbb{R}^{3,1} \times S^1 \times \tilde{S}^1 \times \mathcal{C}_4, \tag{2.30}$$

and add to the strings with momentum and winding (n, w) on S^1 discussed above \tilde{W} $NS5$ -branes wrapped around $S^1 \times \mathcal{C}_4$ and \tilde{N} KK monopoles extended in the same five directions and charged under the Kaluza-Klein gauge field associated with \tilde{S}^1 . The magnetically charged objects are BPS and we consider excitations of this configuration with energy $E \gg M_s$, as before.

To compute the entropy of such states we need the corresponding near-horizon geometry, which is given by

$$\frac{SL(2, \mathbb{R})_k \times U(1)}{U(1)} \times \frac{SU(2)_k}{Z(\tilde{N})_L} \times \mathcal{C}_4. \tag{2.31}$$

Here

$$k = \tilde{N}\tilde{W}, \tag{2.32}$$

and the $U(1)$ quotient acts as in (2.19), (2.20). As before, k (2.32) is the total level of $SL(2, \mathbb{R})$ which receives contributions of $k+2$ and -2 from bosons and fermions respectively. Similarly, for the $SU(2)$ component in (2.31) the total level (2.32) receives a contribution of $k-2$ from a bosonic $SU(2)$ WZW model and $+2$ from three free fermions in the adjoint representation that are needed for superconformal symmetry.

To prove (2.31) one can proceed as follows. For the special case $q_L = q_R = 0$, the background (2.31) reduces to

$$\frac{\mathrm{SL}(2, \mathbb{R})_k}{\mathrm{U}(1)} \times S^1 \times \frac{\mathrm{SU}(2)_k}{Z(\tilde{N})_L} \times \mathcal{C}_4, \quad (2.33)$$

which is known to describe near-extremal $NS5$ -branes and KK monopoles wrapped around $S^1 \times \mathcal{C}_4$ [33, 34]. The $Z(\tilde{N})_L$ quotient in (2.33) is associated with the KK monopoles. It acts holomorphically on the worldsheet; see e.g. [34] for a more detailed discussion. The value of the dilaton at the horizon is determined by the energy density, as in (2.8), [33].

To generalize to the case of non-vanishing electric charges one can perform the sequence of boosts and T-duality on the S^1 discussed for the case of vanishing magnetic charges above. This leads to the background (2.31) whose entropy is given by (2.22), (2.32).

As in the case of vanishing magnetic charges, one can restrict to the special case where M is equal to either $|q_L|$ or $|q_R|$. The two cases are in general inequivalent due to the presence of the KK monopoles, which give rise to the $Z(\tilde{N})$ orbifold in (2.31), which we chose, without loss of generality, to act on the left-moving worldsheet degrees of freedom. The case $M = |q_R|$ corresponds to states that preserve half of the supersymmetry of the $NS5$ — KK system. For $M = |q_L|$ one finds extremal, non-supersymmetric black holes.

In both cases, dimensional reduction of the geometry (2.31) to $3 + 1$ dimensions gives $AdS_2 \times S^2$, with [20, 34]

$$R_{\mathrm{AdS}} = R_{\mathrm{sphere}} = \frac{l_s}{2} \sqrt{k}. \quad (2.34)$$

The dilaton takes the constant value

$$g_4^2 = \sqrt{\frac{\tilde{N}\tilde{W}}{|nw|}}. \quad (2.35)$$

The radii of S^1 and \tilde{S}^1 (2.30) are fixed to the values

$$\frac{R^2}{\alpha'} = \left| \frac{n}{w} \right|, \quad \frac{\tilde{R}^2}{\alpha'} = \frac{\tilde{W}}{\tilde{N}}. \quad (2.36)$$

There are also electric gauge fields on AdS_2 and magnetic gauge fields on S^2 associated with S^1 and \tilde{S}^1 respectively. The entropy (2.22) reduces in this case to

$$S_{\mathrm{extremal}} = 2\pi \sqrt{|nw|\tilde{N}\tilde{W}}. \quad (2.37)$$

In the BPS case, this agrees with the entropy of four charge black holes in $3 + 1$ dimensions (see e.g. [35]).

Further specifying to the case (2.27) leads to the background

$$(AdS_3)_k \times \frac{\mathrm{SU}(2)_k}{Z(\tilde{N})_L} \times \mathcal{C}_4, \quad (2.38)$$

which was studied in [34].

Note that unlike the pure fundamental string case, here all the symmetries of the branes are realized in the near-horizon description. In particular, the $SO(3)$ symmetry corresponding to rotations in \mathbb{R}^3 in (2.30) is realized in $\frac{SU(2)_k}{Z(N)_L}$, and all unbroken supercharges act non-trivially on the background. This is natural, since for large magnetic charges the black holes are large and one can think of (2.31), (2.38) as a supergravity background, while the corresponding solution for strings (2.18), (2.28), is necessarily strongly curved.³

3. Heterotic strings

The worldsheet construction of the heterotic string combines the right-movers of the superstring with the left-movers of the bosonic string. Thus it is useful, as a warm-up exercise, to first study the latter. As usual, due to the infrared instability of the twenty six dimensional flat spacetime background, reflected in the presence of the bosonic string tachyon, this case is not entirely physical, but it is useful as preparation to the heterotic string.

3.1 Thermodynamic description of highly excited bosonic strings

The entropy of highly excited perturbative bosonic string states is linear in the energy, as for type II (2.2). The inverse Hagedorn temperature is given by

$$\beta_H = 4\pi l_s . \tag{3.1}$$

Following the logic of section 2 one might hope that for a particular value of d the near-horizon geometry of highly excited strings in

$$\mathbb{R}^{d-1,1} \times \mathcal{C}_{26-d} \tag{3.2}$$

is given by (compare to (2.12))

$$\frac{SL(2, \mathbb{R})_{k_b}}{U(1)} \times \mathcal{C}_{26-d} . \tag{3.3}$$

Criticality of the background (3.3) implies that

$$\frac{3k_b}{k_b - 2} - 1 = d . \tag{3.4}$$

One can determine k_b and d by requiring that the thermodynamic entropy of the two dimensional black hole (3.3) agree with the microscopic entropy (2.2), (3.1).

The entropy of the bosonic two dimensional black hole (3.3) is again given by (2.10), (2.11) (with $k \rightarrow k_b$). Comparing to (3.1) we conclude that the level k_b must be given by

$$k_b = 4 . \tag{3.5}$$

³A related fact is that the asymptotically linear dilaton throat associated with the strings, (2.12), (2.18), is *not* obtained from the one with non-zero magnetic charges by setting the magnetic charges to zero. Indeed, the background (2.31) – (2.33) only makes sense for $\widetilde{N\tilde{W}} \geq 2$.

Note that while this value looks different from the one we found in the type II case ($k = 2$), it is in fact the same. Before modding out by $U(1)$, we had in the type II case an $SL(2, \mathbb{R})$ WZW model with $k_b = 4$ and three free fermions which contributed -2 to the level, for a total of $k \equiv k_b - 2 = 2$. Here, we have just the bosonic WZW model, whose level is the same as for type II. This fact will play an important role in the generalization to the heterotic string.

Plugging (3.5) into (3.4) we find that in the bosonic string our construction works for $d = 5$, in contrast to the type II case where it worked for $d = 4$ (2.1). This seems like a problem for the heterotic case which combines the two, but as we will see next, there is a natural conjecture that can be made there as well.

3.2 Heterotic strings as black holes

The heterotic string compactified to $4 + 1$ dimensions on a manifold \mathcal{C}_5 is described by the background

$$\mathbb{R}^{4,1} \times \mathcal{C}_5 . \tag{3.6}$$

We would like to find the near-horizon geometry of highly excited perturbative string states in this background. Following the discussion of section 2 and the previous subsection, it is natural to propose that the asymptotic form of this geometry is (compare to (2.6))

$$\mathbb{R}_t \times \mathbb{R}_\phi \times \mathcal{C}_5 . \tag{3.7}$$

The central charge accounting works as follows. For the left-moving (bosonic) sector, the central charge of $\mathbb{R}_t \times \mathbb{R}_\phi$ is given by

$$c = c_t + c_\phi = 1 + 1 + 3Q^2 = 5 , \tag{3.8}$$

where we took the slope of the linear dilaton to be $Q = 1$, as before. Thus, the left-moving central charge of (3.7) is critical.

For the right-moving (fermionic) sector, in addition to the bosonic fields (t, ϕ) we have their two worldsheet superpartners which together with (3.8) bring the central charge to six. However, the total central charge in this sector has to be that of $\mathbb{R}^{4,1}$, which is $15/2$. Thus, we are missing $3/2$ units of c_R .

This is precisely the central charge of three free fermions $\bar{\psi}_i$, $i = 1, 2, 3$, which realize a level two right-moving $SU(2)_R$ current algebra, and an $N = 1$ superconformal algebra needed for consistency of the fermionic string. We add these right-moving free fermions to (3.7) and interpret the resulting $SU(2)$ symmetry as an $SU(2)_R$ subgroup of the $SO(4) = SU(2)_L \times SU(2)_R$ rotation group of \mathbb{R}^4 (3.6). The resulting background is a natural candidate for the asymptotic form of the near-horizon geometry of perturbative heterotic strings in $4 + 1$ non-compact dimensions.

The heterotic background described above is qualitatively different from its type II counterpart. It is $4 + 1$ rather than $3 + 1$ dimensional, and while in the type II case the $SO(3)$ rotation group acts trivially on the near-horizon geometry, in the heterotic string an $SU(2)_R$ subgroup of $SO(4)$ acts non-trivially.

So far we only described the asymptotic form of the near-horizon geometry. The full background is

$$\frac{\text{SL}(2, \mathbb{R})_2}{\text{U}(1)} \times \{\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3\} \times \mathcal{C}_5 . \tag{3.9}$$

The first factor in (3.9) is a heterotic coset CFT with (0, 2) superconformal symmetry, which we describe next. Models of this sort are not well studied, and certainly deserve more attention. We will consider the more general case where the left-moving $\text{SL}(2, \mathbb{R})$ has level k_b , and the right-moving one is a super affine Lie algebra of total level

$$k = k_b - 2 . \tag{3.10}$$

The case (3.9) corresponds to $k = 2$, $k_b = 4$.

One can define the model algebraically, by specifying the current that is being gauged. As in the type II and bosonic discussions above, the left and right moving components of this current are Cartan subalgebra generators of the left and right-moving $\text{SL}(2, \mathbb{R})$, J_3 and \bar{J}_3 . However, in the heterotic case the levels of the left and right-moving $\text{SL}(2, \mathbb{R})$'s are different (they are given by k_b and k , respectively). Hence, the anomaly free current is

$$\left(J_3, \sqrt{\frac{k_b}{k}} \bar{J}_3 \right) . \tag{3.11}$$

The worldsheet superpartner of \bar{J}_3 is gauged as well.

The Euclidean heterotic coset can be described using an analog of the duality of the cigar to Sine-Liouville in the bosonic case [36, 37] and to $N = 2$ Liouville in the fermionic case [38]. We start with the Euclidean version of (3.7), $\mathbb{R}_y \times \mathbb{R}_\phi$ (with $y = it$). The Liouville deformation is a combination of Sine-Liouville for the left-movers, and $N = 2$ Liouville for the right-movers:

$$\delta \mathcal{L} = \lambda \bar{G}_{-\frac{1}{2}} e^{-\frac{1}{Q}(\phi + i\sqrt{\frac{k_b}{k}} y_L + i y_R)} + \text{c.c.} , \tag{3.12}$$

where Q and k are related by (2.9) and $\bar{G}_{-\frac{1}{2}}$ is the right-moving supersymmetry generator. As in the other cases mentioned above [36–38], we expect the heterotic coset (3.11) to be related to the Liouville model (3.12) by strong-weak coupling duality. It would be interesting to explore this duality further.

The entropy of the heterotic coset described above is given by a combination of the fermionic and bosonic ones,

$$S = \pi l_s M \left(\sqrt{k_b} + \sqrt{k} \right) = \pi l_s M \left(\sqrt{k+2} + \sqrt{k} \right) . \tag{3.13}$$

For the case of interest, (3.9), $k = 2$ and the entropy (3.13) agrees with that of free heterotic strings. Thus, (3.9) is a natural candidate for the near-horizon geometry of highly excited heterotic strings in 4 + 1 dimensions (3.6).

3.3 Charged heterotic strings as black holes

If the geometry (3.6) has the form

$$\mathbb{R}^{4,1} \times S^1 \times \mathcal{C}_4, \tag{3.14}$$

one can study the thermodynamics of states with left and right-moving momentum (q_L, q_R) on S^1 as in subsection 2.2. For large oscillator levels N_L and/or N_R , the entropy of free heterotic strings with mass M and charges (q_L, q_R) is given by

$$S = 2\pi\sqrt{2} \left(\sqrt{2N_L} + \sqrt{N_R} \right) = \pi l_s \sqrt{2} \left(\sqrt{2(M^2 - q_L^2)} + \sqrt{M^2 - q_R^2} \right). \tag{3.15}$$

Note the relative factor of $\sqrt{2}$ between the left-moving (bosonic) and right-moving (fermionic) sectors of the theory (compare to (2.17)).

For $q_L = q_R = 0$ we proposed in the previous subsection that the black hole background (3.9) provides a thermodynamic description of these states. Black holes with generic (q_L, q_R) can be obtained from the uncharged one as before, and correspond to the background

$$\frac{\text{SL}(2, \mathbb{R})_2 \times \text{U}(1)}{\text{U}(1)} \times \{\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3\} \times \mathcal{C}_4. \tag{3.16}$$

The charge to mass ratio of the black hole determines the way the $\text{U}(1)$ in the denominator acts on $\text{SL}(2, \mathbb{R})_2 \times \text{U}(1)$ as in subsection 2.2.

For general k , the entropy of the heterotic coset in (3.16) is given by [20]:

$$S = \pi l_s \left(\sqrt{(k+2)(M^2 - q_L^2)} + \sqrt{k(M^2 - q_R^2)} \right). \tag{3.17}$$

For the special case $k = 2$ it agrees with that of free heterotic strings (3.15).

In the supersymmetric extremal case (2.23) and the non-supersymmetric one $M = |q_L|$, one can again formally think about the coset as a sigma model on $AdS_2 \times S^1$, whose properties are given in (2.24) – (2.26) with $k = 2$. A further restriction to the case (2.27) leads to the heterotic string on $\text{SL}(2, \mathbb{R})_2 \times \{\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3\} \times \mathcal{C}_4$, with the three dimensional string coupling given by (2.29).

3.4 States carrying electric and magnetic charges

To add magnetic charges we consider the heterotic string in the 3 + 1 dimensional background (2.30). In general, the two circles S^1 and \tilde{S}^1 combine with the left-moving worldsheet fields into an even, self-dual Narain torus $\Gamma^{2,18}$ (see e.g. [39]). We will restrict to the case where this torus factorizes, $\Gamma^{2,18} = \Gamma^{1,1} \times \Gamma^{1,1} \times \Gamma^{16}$, and the spacetime fields associated with Γ^{16} are not excited. As in subsection 2.3, we will study configurations that contain \tilde{W} NS5-branes and \tilde{N} KK monopoles wrapped around $S^1 \times \mathcal{C}_4$ and charged magnetically under the gauge fields associated with the Neveu-Schwarz B-field and metric on \tilde{S}^1 , respectively.

We are interested in excitations with mass $M \gg M_s$ and momentum and winding (n, w) around the S^1 . The analogous system in type II string theory has the near-horizon

geometry (2.31). This background, properly interpreted (see [34] and the previous subsections), is the near-horizon geometry in the heterotic case too. The level k is given in this case by

$$k = \widetilde{N}\widetilde{W} + 2 . \tag{3.18}$$

For the right-movers this means that the bosonic $SL(2, \mathbb{R})$ and $SU(2)$ sigma models have current algebras of level $k + 2$ and $k - 2$ respectively, as well as worldsheet fermions in the adjoint that complete both levels to k . In the left-moving sector, the fermions are absent, while the bosonic sigma model has the same properties as for the other worldsheet chirality. The $Z(\widetilde{N})$ orbifold acts on the left-moving, bosonic, side. The entropy corresponding to (2.31) is in this case given by (3.17), with the value of k given in terms of the magnetic charges in (3.18).

As in the type II case, there are a number of special cases in which the background (2.31) simplifies. For uncharged excitations ($q_L = q_R = 0$) one finds again the near-extremal fivebrane/KK monopole background (2.33), with the left/right asymmetry discussed in the general case. The entropy (3.17) takes the Hagedorn form (3.13), with $k = \widetilde{N}\widetilde{W} + 2$.

One can also consider the extremal cases $M = |q_R|$, which is BPS, and $M = |q_L|$, which is not. In both cases, the near-horizon geometry (2.31) reduces to $AdS_2 \times S^2$ [34, 40, 20]. The sizes of the anti de-Sitter space and the sphere are again given by (2.34), with the appropriate value of k , (3.18). The four dimensional string coupling takes a form similar to (2.35), with $\widetilde{N}\widetilde{W} \rightarrow \widetilde{N}\widetilde{W} + 2$ (due to the change in level from (2.32) to (3.18)). The radii of S^1 and \widetilde{S}^1 are independent of k and are again given by (2.36).

As in the type II case, the geometric features above are obtained by continuing from a regime in which the gravity approximation is reliable. For small $(n, w, \widetilde{N}, \widetilde{W})$ one should instead study the exact CFT (2.31), however, the gravity analysis is still useful for many purposes. Indeed, the values of the fields found above from the CFT (2.31) agree with those obtained in higher derivative gravity [41–48, 13, 25].

The entropy (3.17) reduces in the extremal cases to:

$$\begin{aligned} M = |q_R| : \quad S_{\text{susy}} &= 2\pi\sqrt{|nw|(\widetilde{N}\widetilde{W} + 4)}, \\ M = |q_L| : \quad S_{\text{non-susy}} &= 2\pi\sqrt{|nw|(\widetilde{N}\widetilde{W} + 2)}. \end{aligned} \tag{3.19}$$

This should be compared to equation (2.37) which holds for both supersymmetric and non-supersymmetric extremal black holes in the type II case. For BPS black holes, (3.19) agrees with previous analyses [41–48, 13, 25]. For the non-BPS extremal black holes, it agrees with [12, 14, 49].

We see that for generic $\widetilde{N}, \widetilde{W}$ the heterotic construction and results are very similar to the type II ones. There is however an important difference. In the heterotic case, setting $\widetilde{N}\widetilde{W} = 0$ in (3.18), the level k takes the value $k = 2$, for which the heterotic version of (2.31) make sense, and it is natural to associate it with the near-horizon background of heterotic

fundamental strings, as was done in the previous subsections.⁴ Moreover, in this case the extremal entropy (3.19), or more generally the non-extremal one (3.17), agrees precisely with that of perturbative heterotic strings with the same quantum numbers, (3.15), as discussed above.

Finally, note that when $\tilde{N} = 0$ the radius of \tilde{S}^1 at the horizon (2.36) goes to infinity, in agreement with the proposal that the corresponding small black holes describe fundamental heterotic string states in 4+1 dimensions.

4. Discussion

In this note we proposed a thermodynamic description of perturbative string states with $M \gg M_s$ in 3 + 1 (4+1) dimensional type II (heterotic and bosonic) string theory in terms of small black holes in an asymptotically linear dilaton spacetime. We showed that these black holes have the correct entropy for both uncharged and electrically charged states and discussed the generalization to non-zero magnetic charges.

Perturbative strings are believed to be well described by weakly coupled string theory in flat spacetime. A natural question is what is the relation of the black hole description proposed here to the standard approach. A possible answer is the following. String perturbation theory is known to break down at high energies. A dramatic manifestation of this is the formation of large black holes at energies above M_s/g_s^2 , but weak coupling techniques are known to break down in scattering at energies large compared to the string scale as well [50, 51]. Therefore, it is natural to expect that many high energy properties of fundamental strings are hard to compute using perturbative string theory, and are instead captured by the linear dilaton throat geometry.

An example is scattering off a highly excited string. Probes with vanishing angular momentum can explore the throat associated with the fundamental strings and the scattering amplitude might receive a contribution from this region. In order to reproduce the black hole result from the perturbative S-matrix one has to sum over contributions in which the massive target is a multi-string state consisting of an arbitrary number of strings. This inclusive process might be hard to study using the standard perturbative approach, but if this is feasible, it would be interesting to compare the results to those obtained in the two dimensional black hole background. This may lead to new insights on the black hole information paradox, the resolution of the black hole singularity and other related issues.

Our discussion also sheds light on the string/black hole correspondence studied in [21 – 24, 52 – 54]. For states with $M \gg M_s/g_s^2$, the gravity approximation is valid and the appropriate thermodynamic description is in terms of large black holes with low Hawking temperature in asymptotically flat spacetime. As the mass decreases, the Hawking temperature increases. In the correspondence region $M \sim M_s/g_s^2$ the temperature reaches the string scale and string corrections to the geometry become significant. Thus, one would expect significant corrections to the thermodynamics of such string size black holes.

⁴According to [34] the heterotic string in the background of \tilde{W} NS5-branes is described by setting $\tilde{N} = 1$ (and not to zero, as one may naively expect). Setting $\tilde{N} = 0$ seems, instead, to correspond to no fivebranes.

Similarly, the thermodynamics of free strings, which is expected to be valid for $M \ll M_s/g_s^2$, is expected to receive large corrections as one approaches the correspondence region, due to gravitational self-interactions of the strings. Nevertheless, the authors of [21–24] pointed out that extrapolating the large black hole and free string results to this regime leads to qualitative agreement. This is known as the string/black hole correspondence principle.

In Euclidean black hole solutions, the time coordinate approaches asymptotically a circle of circumference β . Winding around this circle is not conserved as strings can unwind near the Euclidean horizon. In [52, 53] it was proposed that the sigma model corresponding to such black holes has a non-zero condensate of the closed string tachyon winding around the circle. For large black holes this is a small non-perturbative effect in the black hole sigma model, which influences the physics in a region of size l_s around the Euclidean horizon. However, as the Hawking temperature increases, the effects of this tachyon become more important and eventually, when the Hawking temperature approaches the Hagedorn temperature, the tachyon becomes massless and its fluctuations extend all the way to infinity.

The Minkowski analog of the tachyon condensate is a gas of fundamental strings at the appropriate temperature. The radial extent of the condensate is a measure of the effective size of such strings. For large mass it is small due to the gravitational effects discussed in [21–24]. As the mass decreases, this size grows since gravity becomes weaker.

For masses in the correspondence region $M \sim M_s/g_s^2$ the strings are strongly interacting and hence we expect the temperature to be well below the Hagedorn temperature. Thus, the tachyon is massive and the effective size of generic string states is of order one, as discussed in [21–24]. This region is hard to analyze from both the black hole and fundamental string perspectives.

As the mass continues to decrease below the correspondence region, the temperature continues to grow and eventually, as $g_s^2 M/M_s \rightarrow 0$, it approaches its limiting value — the Hagedorn temperature. In the process, the tachyon in the Euclidean black hole solution becomes lighter, and its condensate extends farther and farther in the radial direction. This leads to a smooth crossover between the black hole and free string behaviors.

For $g_s^2 M/M_s \ll 1$ a linear dilaton throat develops in a string size region around the horizon. As M decreases, this throat becomes larger, and as long as $M \gg M_s$ the string coupling outside the horizon remains small. In the limit $g_s \rightarrow 0$ with M/M_s large but fixed the temperature approaches the Hagedorn temperature, while the part of the geometry corresponding to finite r approaches $\mathbb{R}^3 \times S^1$ (or $\mathbb{R}^4 \times S^1$ in the heterotic case) and decouples from the linear dilaton throat. One can think of the two parts of the geometry as providing the microscopic and thermodynamic descriptions of the relevant states, respectively. In particular, the entropy of perturbative strings is given by the BH entropy of the corresponding two dimensional black hole.

The Euclidean two dimensional black hole, which we proposed as a description of the near-horizon geometry of small black holes, is known to contain a condensate of a tachyon winding around Euclidean time [36–38]. For finite value of the asymptotic string coupling, this geometry should attach smoothly to the flat spacetime at infinity. Thus, it is natural

to expect that, at least for small black holes, the expectation value of the winding tachyon is non-zero at large distances from the horizon as well, in agreement with the proposal of [52, 53]. Decreasing the BH temperature (or increasing the mass towards the large black hole regime) should not change the fact that the tachyon condensate is non-trivial.

Acknowledgments

We thank O. Aharony, A. Dabholkar, O. Lunin, E. Martinec, M. Porrati and G. Veneziano for discussions. This work is supported in part by the BSF — American-Israel Bi-National Science Foundation. AG is supported in part by the Israel Science Foundation, EU grant MRTN-CT-2004-512194, DIP grant H.52, and the Einstein Center at the Hebrew University. The work of DK is supported in part by DOE grant DE-FG02-90ER40560 and the National Science Foundation under Grant 0529954. AG thanks the EFI at the University of Chicago for hospitality. DK thanks the Aspen Center for Physics, Weizmann Institute and Hebrew University for hospitality during the course of this work.

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